Conic optimization with spectral functions on Euclidean Jordan algebras

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# 1 Nonparametric distribution estimation

| | | Hypatia-NF | | | Hypatia-EF | | | MOSEK-EF | | | ECOS-EF | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| cone | $d$ | st | it | time | st | it | time | st | it | timest | it | time | |
| NegRtdet | 500 | co | 11 | 0.0 | co | 14 | 0.3 | co | 8 | 0.2 | co | 22 | 0.3 |
| | 1000 | co | 11 | 0.1 | co | 13 | 1.5 | co | 8 | 0.7 | co | 23 | 6.4 |
| | 2500 | co | 12 | 0.9 | co | 15 | 13. | co | 8 | 9.0 | co | 23 | 107. |
| | 5000 | co | 12 | 6.5 | co | 14 | 68. | co | 8 | 67. | co | 24 | 895. |
| | 10000 | co | 10 | 49. | co | 16 | 446. | co | 8 | 496. | tl | * | * |
| | 15000 | co | 14 | 172. | co | 14 | 1216. | co | 8 | 1589. | sk | * | * |
| | 20000 | co | 15 | 404. | tl | 7 | 1863. | tl | * | * | sk | * | * |
| | 25000 | co | 14 | 756. | sk | * | * | sk | * | * | sk | * | * |
| | 30000 | co | 13 | 1252. | sk | * | * | sk | * | * | sk | * | * |
| NegLog | 500 | co | 17 | 0.1 | co | 11 | 0.3 | co | 8 | 0.1 | co | 18 | 0.3 |
| | 1000 | co | 17 | 0.2 | co | 13 | 1.5 | co | 16 | 0.5 | co | 17 | 11. |
| | 2500 | co | 25 | 1.6 | co | 12 | 11. | co | 8 | 3.5 | co | 18 | 88. |
| | 5000 | co | 29 | 11. | co | 13 | 153. | co | 11 | 22. | co | 18 | 324. |
| | 10000 | co | 36 | 85. | co | 12 | 678. | co | 8 | 134. | tl | * | * |
| | 15000 | co | 40 | 279. | tl | * | * | co | 12 | 440. | sk | * | * |
| | 20000 | co | 46 | 670. | sk | * | * | co | 18 | 1051. | sk | * | * |
| | 25000 | co | 50 | 1331. | sk | * | * | tl | 10 | 1812. | sk | * | * |
| | 30000 | tl | 35 | 1817. | sk | * | * | sk | * | * | sk | * | * |
| NegEntropy | 500 | co | 14 | 0.0 | co | 12 | 0.3 | co | 17 | 0.1 | co | 19 | 0.3 |
| | 1000 | co | 17 | 0.2 | co | 12 | 1.7 | co | 13 | 0.5 | co | 18 | 15. |
| | 2500 | co | 24 | 1.6 | co | 13 | 27. | co | 19 | 4.8 | co | 18 | 44. |
| | 5000 | co | 28 | 11. | co | 12 | 67. | co | 11 | 23. | co | 19 | 361. |
| | 10000 | co | 35 | 84. | co | 12 | 792. | co | 12 | 146. | tl | 7 | 2157. |
| | 15000 | co | 43 | 293. | co | 11 | 1053. | co | 8 | 420. | sk | * | * |
| | 20000 | co | 45 | 663. | tl | 7 | 1861. | co | 15 | 1003. | sk | * | * |
| | 25000 | co | 50 | 1332. | sk | * | * | co | 8 | 1719. | sk | * | * |
| | 30000 | tl | 35 | 1824. | sk | * | * | tl | * | * | sk | * | * |
| NegSqrt | 500 | co | 20 | 0.1 | co | 10 | 0.7 | co | 6 | 0.2 | co | 9 | 0.2 |
| | 1000 | co | 19 | 0.2 | co | 10 | 3.7 | co | 7 | 0.8 | co | 8 | 11. |
| | 2500 | co | 20 | 1.4 | co | 11 | 32. | co | 8 | 14. | co | 8 | 161. |
| | 5000 | co | 22 | 10. | co | 9 | 153. | co | 5 | 54. | co | 8 | 343. |
| | 10000 | co | 25 | 70. | co | 11 | 1026. | co | 7 | 478. | tl | 6 | 2157. |
| | 15000 | co | 27 | 227. | rl | * | * | tl | 6 | 1966. | sk | * | * |
| | 20000 | co | 32 | 555. | sk | * | * | sk | * | * | sk | * | * |
| | 25000 | co | 36 | 1111. | sk | * | * | sk | * | * | sk | * | * |
| | 30000 | tl | 35 | 1821. | sk | * | * | sk | * | * | sk | * | * |

## 2 Experiment design

| | | Hypatia-NF | | | Hypatia-EF | | | MOSEK-EF | | |
|---|---|---|---|---|---|---|---|---|---|---|
| cone | $d$ | st | it | time | st | it | time | st | it | time |
| NegRtdet | 25 | co | 20 | 0.0 | co | 14 | 0.2 | co | 18 | 0.7 |
| | 50 | co | 24 | 0.2 | co | 15 | 2.7 | co | 11 | 8.9 |
| | 75 | co | 18 | 0.3 | co | 15 | 13. | co | 10 | 49. |
| | 100 | co | 20 | 0.6 | co | 17 | 51. | co | 10 | 190. |
| | 150 | co | 19 | 1.6 | co | 17 | 357. | co | 9 | 1178. |
| | 200 | sp | 9 | 2.3 | co | 19 | 1596. | rl | * | * |
| | 300 | co | 19 | 16. | rl | * | * | sk | * | * |
| | 400 | co | 19 | 37. | sk | * | * | sk | * | * |
| | 500 | co | 23 | 95. | sk | * | * | sk | * | * |
| | 600 | co | 22 | 159. | sk | * | * | sk | * | * |
| | 700 | co | 19 | 233. | sk | * | * | sk | * | * |
| | 800 | co | 20 | 417. | sk | * | * | sk | * | * |
| | 900 | co | 18 | 554. | sk | * | * | sk | * | * |
| NegLog | 25 | co | 23 | 0.2 | co | 12 | 0.2 | co | 20 | 0.7 |
| | 50 | co | 24 | 0.5 | co | 13 | 2.4 | co | 11 | 9.0 |
| | 75 | co | 23 | 1.3 | co | 13 | 13. | co | 11 | 54. |
| | 100 | co | 22 | 1.7 | co | 13 | 40. | co | 10 | 201. |
| | 150 | co | 25 | 5.4 | co | 15 | 319. | co | 10 | 1305. |
| | 200 | co | 21 | 10. | co | 14 | 1199. | tl | 0 | 1853. |
| | 300 | co | 29 | 49. | rl | * | * | sk | * | * |
| | 400 | co | 31 | 120. | sk | * | * | sk | * | * |
| | 500 | co | 39 | 297. | sk | * | * | sk | * | * |
| | 600 | co | 41 | 582. | sk | * | * | sk | * | * |
| | 700 | co | 41 | 902. | sk | * | * | sk | * | * |
| | 800 | co | 44 | 1528. | sk | * | * | sk | * | * |
| | 900 | tl | 35 | 1801. | sk | * | * | sk | * | * |
| NegSqrtConj | 25 | co | 26 | 0.2 | co | 14 | 0.2 | sp | 14 | 0.6 |
| | 50 | co | 28 | 0.6 | co | 14 | 2.4 | co | 8 | 7.8 |
| | 75 | co | 32 | 1.7 | co | 15 | 12. | co | 8 | 46. |
| | 100 | co | 36 | 3.3 | co | 16 | 47. | co | 7 | 162. |
| | 150 | co | 46 | 9.0 | co | 19 | 382. | co | 8 | 1170. |
| | 200 | co | 48 | 21. | co | 19 | 1601. | rl | * | * |
| | 300 | co | 57 | 89. | rl | * | * | sk | * | * |
| | 400 | co | 66 | 233. | sk | * | * | sk | * | * |
| | 500 | co | 69 | 479. | sk | * | * | sk | * | * |
| | 600 | co | 73 | 916. | sk | * | * | sk | * | * |
| | 700 | co | 75 | 1527. | sk | * | * | sk | * | * |
| | 800 | tl | 59 | 1811. | sk | * | * | sk | * | * |
| NegPower(1/3) | 25 | co | 19 | 0.2 | sp | 30 | 158. | co | 16 | 3.4 |
| | 50 | co | 26 | 0.6 | rl | * | * | sp | 42 | 225. |
| | 75 | co | 27 | 1.5 | sk | * | * | tl | 41 | 1817. |
| | 100 | co | 24 | 2.1 | sk | * | * | sk | * | * |
| | 150 | co | 29 | 6.3 | sk | * | * | sk | * | * |
| | 200 | co | 27 | 13. | sk | * | * | sk | * | * |
| | 300 | co | 27 | 45. | sk | * | * | sk | * | * |
| | 400 | co | 30 | 119. | sk | * | * | sk | * | * |
| | 500 | co | 27 | 212. | sk | * | * | sk | * | * |
| | 600 | co | 29 | 420. | sk | * | * | sk | * | * |
| | 700 | co | 30 | 697. | sk | * | * | sk | * | * |
| | 800 | co | 28 | 972. | sk | * | * | sk | * | * |
| | 900 | co | 30 | 1541. | sk | * | * | sk | * | * |

# 3 Central polynomial Gram matrix

| | | | Hypatia-NF | | | Hypatia-EF | | | MOSEK-EF | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| cone | $m$ | $k$ | st | it | time | st | it | time | st | it | time |
| NegEntropy | 1 | 15 | co | 14 | 0.1 | co | 19 | 7.5 | co | 11 | 0.4 |
| | 1 | 25 | co | 16 | 0.6 | co | 29 | 200. | co | 17 | 5.7 |
| | 1 | 50 | co | 24 | 5.5 | rl | * | * | co | 25 | 220. |
| | 1 | 75 | co | 23 | 23. | sk | * | * | tl | 23 | 1845. |
| | 1 | 100 | co | 27 | 72. | sk | * | * | sk | * | * |
| | 1 | 125 | co | 29 | 167. | sk | * | * | sk | * | * |
| | 1 | 150 | co | 33 | 426. | sk | * | * | sk | * | * |
| | 1 | 175 | co | 32 | 826. | sk | * | * | sk | * | * |
| | 1 | 200 | tl | 37 | 1807. | sk | * | * | sk | * | * |
| | 4 | 2 | co | 12 | 0.1 | co | 14 | 4.0 | co | 11 | 0.3 |
| | 4 | 3 | co | 17 | 1.1 | tl | 19 | 1885. | co | 22 | 27. |
| | 4 | 4 | co | 23 | 15. | sk | * | * | co | 28 | 1423. |
| | 4 | 5 | co | 28 | 140. | sk | * | * | m | * | * |
| | 4 | 6 | tl | 33 | 1830. | sk | * | * | sk | * | * |
| NegEntropyConj | 1 | 15 | co | 19 | 0.2 | co | 44 | 16. | co | 27 | 1.1 |
| | 1 | 25 | co | 22 | 0.8 | co | 53 | 376. | co | 25 | 8.9 |
| | 1 | 50 | co | 27 | 6.3 | rl | * | * | co | 34 | 316. |
| | 1 | 75 | co | 28 | 28. | sk | * | * | tl | 20 | 1822. |
| | 1 | 100 | co | 32 | 84. | sk | * | * | sk | * | * |
| | 1 | 125 | co | 33 | 188. | sk | * | * | sk | * | * |
| | 1 | 150 | co | 34 | 423. | sk | * | * | sk | * | * |
| | 1 | 175 | co | 35 | 884. | sk | * | * | sk | * | * |
| | 1 | 200 | co | 36 | 1729. | sk | * | * | sk | * | * |
| | 4 | 2 | co | 17 | 0.1 | co | 24 | 6.6 | co | 14 | 0.4 |
| | 4 | 3 | co | 24 | 1.5 | tl | 18 | 1844. | co | 22 | 27. |
| | 4 | 4 | co | 31 | 20. | sk | * | * | co | 31 | 1477. |
| | 4 | 5 | co | 38 | 190. | sk | * | * | m | * | * |
| | 4 | 6 | tl | 34 | 1820. | sk | * | * | sk | * | * |
| Power12(1.5) | 1 | 15 | co | 16 | 0.2 | er | 34 | 14. | co | 15 | 0.6 |
| | 1 | 25 | co | 22 | 0.8 | er | 41 | 291. | co | 19 | 6.7 |
| | 1 | 50 | co | 31 | 7.0 | rl | * | * | co | 24 | 230. |
| | 1 | 75 | co | 38 | 38. | sk | * | * | tl | 22 | 1834. |
| | 1 | 100 | co | 44 | 117. | sk | * | * | sk | * | * |
| | 1 | 125 | co | 42 | 243. | sk | * | * | sk | * | * |
| | 1 | 150 | co | 50 | 649. | sk | * | * | sk | * | * |
| | 1 | 175 | co | 54 | 1400. | sk | * | * | sk | * | * |
| | 1 | 200 | tl | 37 | 1822. | sk | * | * | sk | * | * |
| | 4 | 2 | co | 16 | 0.1 | sp | 28 | 7.7 | co | 13 | 0.4 |
| | 4 | 3 | co | 26 | 1.7 | tl | 18 | 1818. | co | 18 | 24. |
| | 4 | 4 | co | 39 | 25. | sk | * | * | tl | 37 | 1820. |
| | 4 | 5 | co | 47 | 235. | sk | * | * | sk | * | * |
| | 4 | 6 | tl | 33 | 1837. | sk | * | * | sk | * | * |
| Power12Conj(1.5) | 1 | 15 | co | 21 | 0.2 | co | 43 | 16. | co | 20 | 0.8 |
| | 1 | 25 | co | 21 | 0.7 | co | 60 | 412. | co | 22 | 7.7 |
| | 1 | 50 | co | 26 | 5.9 | rl | * | * | co | 27 | 254. |
| | 1 | 75 | co | 28 | 27. | sk | * | * | tl | 20 | 1826. |
| | 1 | 100 | co | 30 | 77. | sk | * | * | sk | * | * |
| | 1 | 125 | co | 32 | 178. | sk | * | * | sk | * | * |
| | 1 | 150 | co | 35 | 430. | sk | * | * | sk | * | * |
| | 1 | 175 | co | 39 | 972. | sk | * | * | sk | * | * |
| | 1 | 200 | tl | 38 | 1821. | sk | * | * | sk | * | * |
| | 4 | 2 | co | 21 | 0.1 | co | 37 | 10. | co | 19 | 0.6 |
| | 4 | 3 | co | 35 | 2.1 | tl | 18 | 1828. | co | 25 | 33. |
| | 4 | 4 | co | 49 | 31. | sk | * | * | co | 32 | 1598. |
| | 4 | 5 | co | 58 | 286. | sk | * | * | m | * | * |
| | 4 | 6 | tl | 34 | 1829. | sk | * | * | sk | * | * |

# 4 Classical-quantum channel capacity

| | Hypatia-NF | | | Hypatia-EF | | | MOSEK-EF | | |
|---|---|---|---|---|---|---|---|---|---|
| $d$ | st | it | time | st | it | time | st | it | time |
| 10 | co | 17 | 0.0 | co | 22 | 2.6 | co | 14 | 0.5 |
| 20 | co | 21 | 0.1 | co | 44 | 688. | co | 19 | 14. |
| 30 | co | 22 | 0.2 | tl | 8 | 1841. | co | 30 | 179. |
| 40 | co | 24 | 0.4 | sk | * | * | co | 29 | 753. |
| 50 | co | 24 | 0.6 | sk | * | * | tl | 20 | 1805. |
| 75 | co | 33 | 1.8 | sk | * | * | sk | * | * |
| 100 | co | 34 | 3.7 | sk | * | * | sk | * | * |
| 150 | co | 40 | 13. | sk | * | * | sk | * | * |
| 200 | co | 43 | 30. | sk | * | * | sk | * | * |
| 250 | co | 42 | 57. | sk | * | * | sk | * | * |
| 300 | co | 47 | 111. | sk | * | * | sk | * | * |
| 350 | co | 47 | 167. | sk | * | * | sk | * | * |
| 400 | co | 49 | 280. | sk | * | * | sk | * | * |
| 450 | co | 53 | 420. | sk | * | * | sk | * | * |
| 500 | co | 49 | 533. | sk | * | * | sk | * | * |
| 550 | co | 53 | 740. | sk | * | * | sk | * | * |
| 600 | co | 58 | 1105. | sk | * | * | sk | * | * |
| 650 | co | 58 | 1463. | sk | * | * | sk | * | * |
| 700 | co | 59 | 1748. | sk | * | * | sk | * | * |
| 750 | tl | 49 | 1805. | sk | * | * | sk | * | * |